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# NOTE

## A Sufficient Condition for Hamiltonian Connectedness

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In a graph $G$, a cycle (path) containing all the vertices of $G$ is called a *Hamiltonian cycle* (*path*). The graph $G$ is said to be *Hamiltonian* if it has a Hamiltonian cycle. In [2], Ore defined a graph $G$ to be *Hamiltonian connected* if each pair of vertices of $G$ is joined by a Hamiltonian path. The degree $d(v)$ of a vertex $v$ of $G$ is the number of edges incident with $v$. Let $D$ be the $n$-tuple $(d_1, d_2, ..., d_n)$ of non-negative integers with $d_1 \leqslant d_2 \leqslant \cdots \leqslant d_n$. A graph $G$ with $n$ vertices is said to be a *realization* of $D$ if the vertices of $G$ can be arranged so that $d(v_i) = d_i$, $i = 1, 2, ..., n$. Let $\Gamma(D)$ be the set of all graphs which are realizations of $D$. Bondy [1] proved the following sufficient condition that a graph be Hamiltonian: *Let $D$ satisfy the following condition:*

$$d_k \leqslant k, \qquad d_l \leqslant l(k \neq l)) \Rightarrow d_k + d_l \geqslant n, \tag{A}$$

*where $n \geqslant 3$. Then, if $G \in \Gamma(D)$, $G$ is Hamiltonian.*

Ore [2] proved some conditions that ensure that a graph be Hamiltonian connected. This note gives an analog to the result of Bondy for Hamiltonian connectedness.

THEOREM. *Let $D$ satisfy the condition:*

$$d_k \leqslant k + 1, \qquad d_l \leqslant l + 1(k \neq l) \Rightarrow d_k + d_l > n. \tag{B}$$

*If $G \in \Gamma(D)$, then $G$ is Hamiltonian connected.*

*Proof.* To show that $G$ is Hamiltonian connected, let $u$ and $v$ be any two vertices of $G$. Define a new graph $H$ from $G$ by adding a new vertex $w$, an edge joining $w$ to $u$, and an edge joining $w$ to $v$. Since the degree sequence of $G$ satisfies condition (B), the degree sequence of $H$ satisfies condition (A), and, by the result of Bondy [1], $H$ has a Hamiltonian cycle. Therefore $G$ has a Hamiltonian path joining $u$ and $v$, and $G$ is Hamiltonian connected.

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## References

1. J. A. Bondy, Properties of graphs with constraints on degrees, *Studia Scientiarum Mathematicarum Hungarica*, **4** (1969), 473-475.
2. O. Ore, Hamiltonian connected graphs, *J. Math. Pures Appl.* **42** (1963), 21–27.